Solutions to Workbook-2 [Mathematics] | Permutation & Combination

Level - 2	DAILY TUTORIAL SHEET 6

126.(A) No. of even subsets = ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = \dfrac{2^n}{2} = 2^{n-1}$

[Use $2^{n-1} = {}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots$] [Refer Binomial theorem]

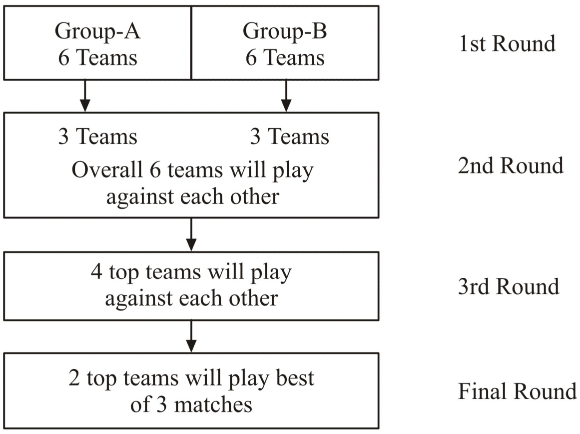
127.(A) Let no. of participants at the beginning were n.
Now, left players are $n-2$

According to question: ${}^{n-2}C_2 + 3 + 3 = 84$

$\dfrac{(n-2)(n-3)}{2} = 78 \Rightarrow n^2 - 5n - 150 = 0 \Rightarrow (n-15)(n+10) = 0 \Rightarrow n = 15$ is the only solution.

128.(B)

Flow - Chart



Round - 1

No. of matches $\equiv {}^6C_2$ ways $+ {}^6C_2$

Round - 2

No. of matches $\equiv {}^6C_2$ ways

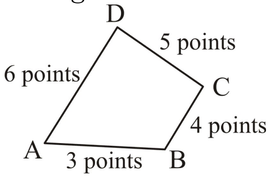
Round - 3

No. of matches $\equiv {}^4C_2$ ways

Final Round
No. of matches $\equiv 2$ for minimum matches to be played.

Total no. of matches $= \left({}^6C_2 + {}^6C_2\right) + {}^6C_2 + {}^4C_2 + 2 = 15 + 15 + 15 + 6 + 2 = 53$

129.(D) No of triangle can be found as:



${}^3C_1\,{}^4C_1\,{}^5C_1 + {}^3C_1\,{}^4C_1\,{}^6C_1 + {}^3C_1\,{}^5C_1{}^6C_1 + {}^4C_1{}^5C_1{}^6C_1 = 60 + 72 + 90 + 120 = 342$

No. of possible triangles = 342

130.(C) To form pairs, select 2 digits from 4 digits in ${}^4C_2$ ways and select 2 other digits in ${}^2C_2$ ways.

Then arrange then in $\dfrac{6!}{2!2!}$ ways

Number of numbers $= {}^4C_2 \times {}^2C_2 \times \frac{6!}{2!2!} = 1080$

131.(D) $x = \sum_{k=1}^{100} k!$

$x = 1! + 2! + 3! + 4! + 5! + 6! + 7! + 8! + 9! + 10! + ... + 100!$

$= (01) + (02) + (06) + (24) + (120) + (720) + ...$

We are only interested in last two digits

After 5! last term is zero and after 10! last two terms are zero.

So, sum of last two digits $\equiv 01 + 02 + 06 + 24 + 20 + 20 + 40 + 20 + 80$

Hence last two digits = 13

132.(C) Voter can vote in ${}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_{n-1}$ ways $= 2^n - {}^nC_0 - {}^nC_n = 2^n - 2 = 62 \Rightarrow n = 6$

133.(A) Number of ways in which person predicts the outcome is correct is

${}^{20}C_{10} 1^{10}$

└→(as outcome is correct)

Rest 10 predictions are wrong and hence can be given in 2^{10} ways.

Total ways $= {}^{20}C_{10} \times 2^{10}$.

134.(B) Select 2 places for digits 2 in ${}^7C_2$ ways and arrange them in 1 way. Remaining 5 places can be filled in 2^5 ways as each place can hold 1 or 3.

Hence, total number of ways $= {}^7C_2 \times 2^5$

135.(C) $x_1 + x_2 + x_3 = 11$ where $1 \le x_i \le 6 \forall i = 1$ to 3 [Assume dice are identical]

Number of ways = coeff. of x^{11} in $\left(x + x^2 + ... + x^6\right)^3$

= coeff. of x^8 in $\left(1 + x + x^2 + ... + x^5\right)^3$ = coeff. of x^8 in $\left(\frac{1-x^6}{1-x}\right)^3$

= coeff of x^8 in $\left({}^3C_0 - {}^3C_1 x^6\right)(1-x)^{-3}$ = coeff of x^8 in $(1-x)^{-3} - 3$ coeff of x^2 in $(1-x)^{-3}$

$= {}^{3+8-1}C_8 - 3\ {}^{3+2-1}C_2 = {}^{10}C_2 - 3\,{}^4C_2 = 45 - 18 = 27$

136.(B) The mint has to perform two jobs, viz.

(i) Selecting the number of days in the February month (there can be 28 days or 29 days).

(ii) Selecting the first day of the February month.

The first job can be completed in 2 ways while the second can be performed in 7 ways by selecting any one of the seven days of a week.

137.(D) We have $E = \frac{31!}{2^{31}(32!)} = \frac{1}{2^{31}(32)} = \frac{1}{2^{36}} = 2^{-36} = \left(2^3\right)^{-12} = 8^{-12}$ Thus, $x = -12$

138.(B) Since a multiple-choice question can be answered in $\left(2^3 - 1\right)$ ways there are 7 ways of answering each of the 4 questions. So, total number of different sequences of answers $7^4 = 2401$

But no student has written all the correct answers and different students have given different sequences of answers.

So, Maximum number of students in the class = Number of sequences except one sequence in which all answers are correct = $2401 - 1 = 2400$

139.(B) The number of triangles that can be formed by using the vertices of a regular polygon of n sides is ${}^nC_3$. That is, $T_n = {}^nC_3$

Now, $T_{n+1} - T_n = 21 \Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21 \Rightarrow {}^nC_2 + {}^nC_3 - {}^nC_3 = 21$ $\left[\because {}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r\right]$

$\Rightarrow \quad \frac{1}{2}n(n-1) = 21 \Rightarrow n = -6$ or 7 As n is a positive integer, $n = 7$.

140.(D) Number of groups having 4 boys and 1 girl $= \left({}^{4}C_4\right)\left({}^{g}C_1\right) = g$

and number of groups having 3 boys and 2 girls $= \left({}^{4}C_3\right)\left({}^{g}C_2\right) = 2g(g-1)$

Thus, the number of dolls distributed $= g(1) + (2)\left[2g(g-1)\right] = 4g^2 - 3g$.

We are given $4g^2 - 3g = 85 \Rightarrow g = 5$